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AUTHOR Henderson, George L.; And Others
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ABSTRACT

Overall goals for a model K-8 mathematics program are stated. A hierarchy of over 400 mathematics content objectives for grades K-8 are listed in a prerequisite and sequential order and also organized in a grid form. Suggestions as to how the objectives can be used and a checklist of objectives upon which Wisconsin's statewide mathematics assessment test items will be based are included. (DT)

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WISCONSIN STATEWIDE ASSESSMENT

MATHEMATICS

AN EXEMPLARY MATHEMATICS PROGRAM

GRADES K THROUGH 8

AND

A HIERARCHY OF

STUDENT BEHAVIORAL OBJECTIVES

K-8

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F O R E W A R D

Persons interested in Wisconsin's statewide assessment program in mathematics will find the information in this paper helpful as they seek specifics regarding the elementary mathematics program, its goals and objectives, as determined by a statewide group of mathematics educators.

A reasonable assessment of pupil achievement cannot be made unless specific goals and objectives have first been identified. We are greatly indebted to the many people who served selflessly and helpfully on our committees.

George L. Henderson
Consultant-Mathematics Education
State of Wisconsin
Department of Public Instruction

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WISCONSIN MATHEMATICS ASSESSMENT ADVISORY COMMITTEE

CHAIRMAN: Mr. George L. Henderson*
Consultant-Mathematics Education
Department of Public Instruction
126 Langdon Street
Madison, Wisconsin 53702

Mr. John Aceto
K-12 Mathematics Coordinator
Unified School District #1
2230 Northwestern Avenue
Racine, Wisconsin 53404

Mrs. Marjorie Bretschneider
Spring Road Elementary School
1191 Winchester Road
Neenah, Wisconsin 54956

Mrs. Ruth Brusewitz
Hortonville Community Schools
246 North Olk Street
Hortonville, Wisconsin 54944

Dr. Arnold Chandler
Administrator
Instructional Specialists Section
Department of Public Instruction
126 Langdon Street
Madison, Wisconsin 53702

Dr. Roland W. Christensen
Mathematics Department
1033 Cowley Hall
University of Wisconsin
La Crosse, Wisconsin 54601

Mr. Peter Christiansen III
K-12 Mathematics Coordinator
Madison Public Schools
545 West Dayton Street
Madison, Wisconsin 53703

Dr. C. Patrick Collier
Mathematics Department
University of Wisconsin
Oshkosh, Wisconsin 54901

Mr. Dale Durnford
Principal
Robinson Elementary School
Cranston Road
Beloit, Wisconsin 53511

Miss Helga Eggener
Elementary Supervisor
Appleton Public Schools
120 East Harris Street
Appleton, Wisconsin 54911

Mr. Al Epton
Canterbury Elementary School
7000 Enfield Avenue
Greendale, Wisconsin 53129

Mr. Lowell Glunn*
Mathematics Department
Monona Grove High School
4400 Monona Drive
Monona, Wisconsin 53716

Mr. Paul Hagerty
Mathematics Curriculum Specialist
Milwaukee Public Schools
P. O. Drawer 10K
Milwaukee, Wisconsin 53208

Mrs. Audrey Hains*
Bose Elementary School
1900 15th Street
Kenosha, Wisconsin 53140

Miss Adeline Hartung
Elementary Supervisor
Milwaukee Public Schools
P. O. Drawer 10K
Milwaukee, Wisconsin 53208

Mr. David Johnson
Chairman, Mathematics Department
Nicolet High School
6701 North Port Washington Road
Milwaukee, Wisconsin 53217

Miss Mary Kay Johnson
McKinley Elementary School
1125 East Taft Street
Appleton, Wisconsin 54911

Mr. Clayton Jorgensen
Teacher
Grantsburg Elementary School
Grantsburg, Wisconsin 54840

Mrs. Glen W. Kessler
Elementary Teacher
Forrest Street E ementary School
Route 4
Black River Falls, Wisconsin 54615

Mrs. Patricia Koster
Jackson Elementary School
1900 West Zinda Drive
Stevens Point, Wisconsin 54481

Mr. Paul Klancher
Teacher
Rice Lake Middle Junior High School
204 Cameron Road
Rice Lake, Wisconsin 54868

Mrs. Patsy R. Lautenschlager
K-12 Mathematics Coordinator
Fond du Lac Public Schools
101 Morningside Drive
Fond du Lac, Wisconsin 54935

Dr. Walter Leffin*
Mathematics & Education Departments
University of Wisconsin
Oshkosh, Wisconsin 54901

Mr. Terrance O. Leigh
Mathematics Department
Univ. of Wisconsin - Green Bay
Marinette County Campus
Bay Shore
Marinette, Wisconsin 54143

Mr. Ted E. Losby
Mathematics Department
J. Madison Memorial High School
201 South Gammon Road
Madison, Wisconsin 53705

Mr. Dennis Mancusi
Bose Elementary School
1900 15th Street
Kenosha, Wisconsin 53140

Mr. Leonard G. Marko
Nicolet Elementary School
Ahnaip Street
Menasha, Wisconsin 54952

Mrs. Joan Martin*
Elementary Teacher
Cudahy Public Schools
3744 East Ramsey Avenue
Cudahy, Wisconsin 53110

Mr. William Miller*
Mathematics Department
John Muir Middle School
1400 West Stewart Avenue
Wausau, Wisconsin 54401

Mr. Dick Peper
Mathematics Department
Grantsburg High School
Grantsburg, Wisconsin 54840

Miss Faustine Perham*
Mathematics Department
Marquette University
Milwaukee, Wisconsin 53233

Mrs. Glenn Pfisterer
Brodhead Elementary School
600 21st Street
Brodhead, Wisconsin 53520

Dr. Tom Romberg*
University of Wisconsin
Research & Development Center
1404 Regent Street
Madison, Wisconsin 53706

Dr. Carroll Rusch
Mathematics Department
Sundquist 334
University of Wisconsin
Superior, Wisconsin 54881

Dr. Jack Schmidt
Department of Public Instruction
126 Langdon Street
Madison, Wisconsin 53702

Mr. Joe Slaney
Summit Elementary
36316 Valley Road
Oconomowoc, Wisconsin 53066

Dr. Henry D. Snyder
Mathematics & Education Departments
University of Wisconsin
Milwaukee, Wisconsin 53201

Mrs. Yvonne Sokolski
Mathematics Teacher
Wheatland Center Elementary School
Route 4, Box 350
Burlington, Wisconsin 53105

Mr. DeLoyd Stertz*
Mathematics Department
University of Wisconsin
Eau Claire, Wisconsin 54701

Mr. Dean E. Stowers
Mathematics Departments
Nicolet College and Technical Inst.
P. O. Box 518
Rhinelander, Wisconsin 54501

Miss Patricia M. Trebatoski
Teacher
Jackson Elementary School
1900 West Zinda Drive
Stevens Point, Wisconsin 54481

Miss Mary Van Beck*
Mathematics Department
Nicolet High School
6701 North Port Washington Road
Milwaukee, Wisconsin 53217

Sister Mary Petronia Van Straten
SSND, Mathematics Department
Mount Mary College
Milwaukee, Wisconsin 53222

Mr. Kenneth Wald
Principal
Cochrane-Fountain City Elementary
Fountain City, Wisconsin 54629

Mr. Bob Wenninger
West De Pere Elementary School
665 Grant Street
West De Pere, Wisconsin 54178

Dr. Bruce Williamson
Mathematics Department
University of Wisconsin
River Falls, Wisconsin 54022

Dr. Margaret Woods
Education Department
University of Wisconsin
Platteville, Wisconsin 53818

Dr. Karl Zahn*
Education Department
University of Wisconsin
Whitewater, Wisconsin 53190

Mr. Donald Zalewski
Department of Public Instruction
126 Langdon Street
Madison, Wisconsin 53702

* Writing Committee

AN EXEMPLARY MATHEMATICS PROGRAM, GRADES K-8

The Wisconsin Mathematics Assessment Advisory Committee subscribes to the following as a description of an exemplary mathematics program for the schools of the state, grades kindergarten through eight:

I. Output characteristics (no order of importance implied):

A. In an ideal program the most important goal of mathematics instruction is the development of the students' ability to solve problems. By a problem we mean that a question is posed for which an answer is not immediately available and the student has a desire to find a solution. It is assumed that the student is required to use more than immediate recall or previously established patterns of action to find a method for arriving at a conclusion. In particular, the skills of mathematics and the correct applications of mathematical principles and methods are essential to this process. Emphasis should be placed on those problems which arise from real life settings.

B. Students should have the facility to apply previously learned mathematical principles, skills and methods to real life situations so that some problems do not arise.

C. The student should attain the prerequisites for continued formal study (and/or independent study) in mathematics, related fields and other subjects using mathematics.

D. The student should realize that while mathematics can be a useful tool for solving problems and making decisions, it should also be recognized as a science, an art form and a language. As a science the student is involved with a study of patterns and also looks at mathematics as a unified structure. As an art form, the student is given the opportunity to be both creative and original. The language of mathematics enables the student to comprehend and communicate ideas which would otherwise be beyond his reach.

E. The program maintains the natural interest young children have in mathematics with the resulting desire to continue study. The student recognizes mathematics as an alive, enjoyable, game-like discipline with profound implications for the future of society.

II. Process characteristics of an ideal exemplary elementary mathematics program:

A. Those characteristics related to the means of achieving the output goals are centered around the notion of success-oriented instruction. Individual pupil success in the

learning of mathematics can occur only when differences between pupils are taken into account. Instruction should be individualized along three dimensions: time, content, and mode. The latter dimension must feature active learner involvement, initiating with concrete manipulative experiences and progressing, as "readiness" permits, to activities dealing with concepts of a more abstract nature. The self-study mode must not dominate at the elementary level. Mathematics should be "talked" as well as "worked," through liberal teacher-pupil and pupil-pupil interaction via group activities and discussion. Cyclic reinforcement and/or content spiraling must be provided as necessary.

B. The ideal elementary mathematics program possesses a built-in accountability system capable of retrieving diagnostic information on individual pupil progress and continuously assessing overall program adequacy and efficiency.

C. In the final analysis, however, the teacher remains the most important instructional variable. Teacher subject matter competence and ability to interact effectively with pupils are indispensable. The ideal elementary mathematics program provides for ongoing teacher training consistent with current trends and research results.

MATHEMATICAL CONTENT OBJECTIVES

Skills are distinct from problem solving. The committee has produced and listed in a prerequisite and sequential order, a hierarchy of mathematics content objectives for grades K-8. An identifying code of the 23 strands (A through Z) is listed on page 8 followed by a grid on the next three pages that combines the 23 strands in columns with 115 horizontal levels. Over 400 content objectives are listed on the following pages.

Using the grid and objectives

Each entry in the grid represents a specific objective in the hierarchy which is coded by a number (row) and a capital letter (column). For example, the first objective in the hierarchy is coded 1Q and is represented by the lower case q in row 1, column Q. The objectives in each column of the grid represent a prerequisite order for presenting the

objectives. For example, objective 5A is a prerequisite for 6A, 6A is a prerequisite for 7A, and so on. The objectives that appear in the same row are judged to be about the same level of difficulty but are not prerequisite for one another. For example, objectives 19C, 19L, and 19Q, can be presented at about the same time, in any order, provided all of the column prerequisites for these objectives have been met.

The master list of objectives, and the accompanying grid, can be a versatile teaching aid for teachers who are attempting to individualize instruction. Material in a basic textbook series can be correlated to the objective hierarchy. The list of objectives can be used as a reference for checking to see if existing programs deal properly with prerequisites. Compendiums of instructional materials can be cross-referenced with the list of objectives. Diagnostic tests, based on the objectives, can be used for individual and group placement of students in an instructional program. Statewide testing will be based on objectives found in this hierarchy. Problem solving items will also be included.

Planned review and reinforcement activities can be organized to include necessary prerequisite skills and knowledge. The hierarchy grid can be used as a chart for recording student progress.

The "spiraling" of learning (review and reinforcement and expansion of previously introduced concepts and skills) is very important for "mastery," but the hierarchy of objectives does not outline a spiral. Also, it is not intended that the hierarchy be considered as the only "correct" prerequisite arrangement of objectives. The objectives listed should be regarded as a suggested guide for introducing various topics and is not intended to be all-inclusive. The teacher may find it necessary to alter the order of the objectives to meet the needs of the individual child in the classroom.

CODE FOR HIERARCHY OF
MATHEMATICS CONTENT OBJECTIVES
K-8

A sets	N ratios
B counting	P 3-dimensional figures
C numeration	Q 2-dimensional figures
D addition	R similarity and congruence
E subtraction	S symmetry
F multiplication	T "money" and "time"
G division	V measurement
H fractions	W number line
J number theory and order	X constructions
K integers	Y ordered pairs
L writing sentences	Z probability and statistics
M solving sentences	

1 through 115 are horizontal "levels"

A through Z are column headings

WISCONSIN STATEWIDE MATHEMATICS
ASSESSMENT TEST ITEMS WILL BE
BASED ON THE FOLLOWING OBJECTIVES*

Column	End-Of-Year Third Grade	End-Of-Year Seventh Grade
A	16-39	91-106
B	16-43	none
C	18-46	62-101
D	16-35	74-100
E	16-40	73-100
F	37-53	74-113
G	45-47	65-102
H	17-45	97-103
J	18-43	64-98
K	none	78-112
L	18-49	68-83
M	23-49	69-106
N	44-46	65-98
P	none	60-106
Q	18-44	70-100
R	44 only	62-103
S	44 only	80-97
T	none	none
V	40-50	61-102
W	18-36	84-102
X	none	76-101
Y	none	97-103
Z	50 only	97-106

Note: Problem solving will also be tested.

* From: K-8 Hierarchy of Mathematics Content Objectives

HIERARCHY OF MATHEMATICS CONTENT OBJECTIVES K-8

	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	V	W	X	Y	Z
1															q								
2															q								
3															q				v				
4															q								
5	a													p	q			t					
6	a													p					v				
7	a																						
8	a	b																t					
9	a	b	c						j									t					
10		b	c																				
11	a		c																				
12		b	c					h			l												
13			c	d																			
14					e																		
15			c								l												
16	a	b	c	d	e						l	m								w			
17			c	d	e			h															
18		b	c						j		l				q					w			
19			c								l				q								
20			c												q								
21			c	d					j		l												
22			c		e						l	m								w			
23					e						l	m											
24											l	m											
25				d								m											
26				d	e													t					
27				d														t					
28				d																			
29				d																			
30			c	d											q								
31			c	d	e										q								
32			c	d	e										q								
33				d	e																		
34				d	e																		
35	a			d	e													t					
36			c			f														w			
37			c			f						m							v				
38										l	m								v				
39	a					f		h		l									v				

HIERARCHY OF MATHEMATICS CONTENT OBJECTIVES K-8

	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	V	W	X	Y	Z
40					e	f													v				
41						f																	
42									j									t					
43		b						h	j														
44			c			f			j				n		q	r	s						
45			c			f	g	h			l	m	n				s						
46			c			f	g			k	l		n						v				
47						f	g				l	m							v				
48						f	g				l									w			
49						f	g				l	m								w			
50						f	g				l								v				z
51						f							n						v				
52						f																	
53				d		f		h												w			
54						f	g	h															
55			c			f																	
56			c			f	g												v				
57						f	g		j										v				
58						f	g		j														
59						f	g																
60						f								p	q	r			v				
61						f	g							p		r			v				
62			c			f	g									r			v				
63						f							n						v				
64			c				g		j				n										
65			c				g						n										
66													n										
67								h	j				n										
68			c						j		l												
69			c							k		m			q					w			
70				d	e	f								p	q								
71															q								
72				d	e										q								
73			c	d	e										q	r							
74				d		f									q								
75					e	f	g									r					x		
76							g														x		
77				d		f	g			k									v		x		

HIERARCHY OF MATHEMATICS CONTENT OBJECTIVES K-8

	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	V	W	X	Y	Z
78					e	f	g			k									v				
79					e		g			k						r			v				
80			c	d		f	g										s		v	w			
81				d		f							n	p	q								
82				d	e						l		n	p	q								
83						f					l		n	p									
84							g						n	p						w			
85						f	g						n	p									
86						f						m							v				
87			c			f							n						v				
88				d															v		x		
89			c	d																	x		
90			c																		x		
91	a		c																v		x		
92	a													p					v		x		
93	a																		v				
94	a																s		v				
95	a																		v				
96	a																		v				
97	a		c	d		f		h	j			m	n		q	r	s		v			y	z
98	a		c	d		f		h	j				n		q	r			v	w	x	y	z
99	a		c					h	j					p	q	r			v		x	y	z
100			c	d	e			h				m			q	r			v		x	y	z
101			c			f	g	h								r			v		x	y	z
102							g	h								r			v	w			z
103								h								r			v	w		y	z
104																			v				z
105																			v				z
106	a		c									m		p	q	r	s		v		x		z
107			c		e							m		p	q	r			v		x		z
108			c	d	e										q	r			v				z
109			c			f	g									r			v	w		y	z
110			c					h				m	n			r			v				z
111			c													r			v	w			z
112			c	d		f				k		m				r			v				
113			c			f									q	r					x	y	
114																			v			y	
115																			v			y	

The student should be able to:

1Q. Recognize closed and open simple curves.

2Q. Recognize the inside and outside of simple closed curves.

3Q. Recognize squares, rectangles, circles, and triangles.

3V. Use appropriately such words as longer, shorter, heavier, lighter, higher, lower, larger, smaller.

4Q. Use the terms inside, outside, and on as related to rectangles, circles and squares.

5A. Determine whether two sets are equivalent by placing the members of the sets in one-to-one correspondence.

5P. Observe distinguishing features of spheres, rectangular prisms (boxes), cylinders, and other objects.

5Q. Recognize that squares, rectangles, triangles, and circles are closed curves and tell whether a point is inside, outside, or on such a curve.

5T. Recognize pennies, nickels, dimes.

6A. Use such terms as more than, as many as, fewer than when comparing sets.

6P. Use the terms round, face, edge, corner, and surface.

6V. Determine which of two line segments is the longer or the shorter, or whether they are the same length.

7A. Compare two non-matching sets of less than 20 objects to decide which set has more (fewer) members.

8A. Utilize the idea "one more than" in organizing sets in natural order.

8B. Identify, without counting, the number of elements in sets with zero, one, two, or three objects and select the set of objects associated with a number zero, one, two, or three given orally.

8T. Recognize the comparative value of coins (pennies, nickels, dimes).

9A. Insert missing sets, such as a set of 3 elements between a set of 2 elements and a set of 4 elements.

9B. Count the members of a set containing ten or fewer members.

9C. Select the numeral that represents the number of objects in a set (containing one to nine members).

9J. Use ordinal numbers through fifth.

9T. Make comparisons in time and count whole units of time (day, week, month, year).

The student should be able to:

10C. Recognize "0" as the symbol for the number of elements in the empty set.

11A. Rearrange sets of objects to demonstrate the joining and separating of sets, and thereby develop a readiness for addition and subtraction.

11C. Determine that 8 is greater than 5 and 5 is less than 8 by comparing appropriate sets of objects and do this for any two numbers less than 10.

12B. Read number words through ten.

12C. Identify the numerals 0 through 10.

12H. Demonstrate one-half, one-fourth of a physical unit.

12L. Use objects to represent the "action" or the conditions of problems given orally, and tell stories about the problem situations.

13C. Use the terms greater than and less than, and equals in sentences. For example: five is greater than two.

13D. Identify the process of addition through experience with joining two sets of objects.

14E. Identify the process of subtraction through experience with separating a subset from a set of objects.

15C. Interpret the place-value concept for writing whole numbers through 99; such as, 89 is the same as 8 tens, 9 ones.

15L. Use the symbols + and = to form sentences such as $3 + 6 = _____$.

16A. Demonstrate with sets of objects the relationship between such sentences as $4 + 2 = 6$, $6 - 2 = 4$, and $6 - 4 = 2$.

16B. Count by 2's, 5's, and 10's.

16C. Name successors and predecessors of each number through 99.

16D. Recognize examples of the commutative property for addition in the set of whole numbers.

16E. Recognize that subtraction is not commutative.

16L. Use the symbols - and = to form sentences such as $6 - 3 = _____$.

16M. Find the solution for sentences like $3 + 4 = \square$ and $5 - 2 = \square$.

16W. Use the number line to illustrate counting by 2's, 5's and by 10's.

17C. Insert missing numbers between two given numbers. For example, list in proper sequence (order) the numbers between 12 and 16.

17D. Recognize zero's property for addition in the set of whole numbers.

The student should be able to:

17E. Recognize zero's special role in subtraction.

17H.-(A) Identify 1/2, 1/4, 1/3 of a
17H.-(B) whole by using physical objects, and associate with numerals.

18B. Determine the cardinality of a set through 999 when the objects are already grouped. For example, 4 bundles of 100 sticks, 9 bundles of 10 sticks, and 4 sticks are the same as 494 sticks.

18C. To demonstrate place value through 999; for example, use bundles of sticks to show 234 = 200 + 30 + 4.

18J. Use ordinal numbers beyond fifth (5th).

18L. Use symbols $<$, $>$, and = in mathematical sentences.

18Q. Recognize physical representations of points and line segments.

18W. Recognize that a point on a line can be described by a number (coordinate).

19C. Read and write any numeral through 999.

19L. Use objects to represent the "action" or conditions of a problem.

19Q. Recognize a point as a position and a line segment or curve as a set of points.

20C. Write three-digit numerals in expanded form; for example, 756 = 700 + 50 + 6.

20Q. Recognize a straight line as a set of points with no beginning and no end.

21C.-(A) Use physical objects to demon-
21C.-(B) strate regrouping; for example, 2 bundles of 10 sticks and 16 sticks have the same number of sticks as 3 bundles of 10 sticks and 6 sticks. Also 4 bundles of 10 sticks and 8 sticks have the same number of sticks as 3 bundles of 10 sticks and 18 sticks.

21D. Recall the addition facts through the sum 18.

21J. Discover from the addition table number patterns through the sum 18.

21L. Write an appropriate mathematical sentence like 3 + 4 = $\square$ or 5 - 2 = $\square$ for physical, pictorial and verbal situations where the "action" of the problem suggests the operation of addition or subtraction.

22C.-(A) Determine greater than, less
22C.-(B) than, and betweenness for
22C.-(C) numbers through 999.

22E. Recall the subtraction facts for sums through 18.

22L. Use many different kinds of placeholders like $\square$, $\triangle$, n, in mathematical sentences.

22M. Find solutions for sentences like $\square + \triangle = 7$, in which many correct solutions are possible.

The student should be able to:

22W.-(A) Use the number line to
22W.-(B) illustrate addition and subtraction problems.

23E. Subtract a one-digit number from a one-digit number.

23L. Make up a problem situation to fit a given mathematical sentence involving addition or subtraction.

23M.-(A) Use sentences like $5 + \square = 12$, $\square + 6 = 8$, $12 - \square = 8$, and $\square - 5 = 6$ to
23M.-(B) represent physical, pictorial and verbal situations and find solutions for the sentences.

24L. Use equivalent sentences like $3 + \square = 7$ and $7 - 3 = \square$ to show subtraction as the inverse of addition.

24M.-(A) Place the correct symbol
24M.-(B) (+ or -) in the placeholder in sentences like $13 \triangle 5 = 8$ and $4 \square 3 < 5$.

25D. Use the associative property of addition in the set of whole numbers; for example, $(3 + 4) + 6 = 3 + (4 + 6)$.

25M.-(A) Find solutions for sentences
25M.-(B) like $3 + 2 = 8 - \square$, $\square + 5 = 8 + 7$, $8 + \square < 12$, and $4 + 9 > \square + 5$, with the aid of sets of objects or the number line.

26D. Add a one-digit number to a two-digit number without regrouping.

26E. Subtract a one-digit number from a two-digit number, without regrouping.

26T. Make change correctly for quantities up to 25¢.

27D. Use the vertical algorithm in the addition of three addends with one-place numerals; for example:

$$\begin{array}{r} 3 \\ 4 \\ +\ 5 \\ \hline \end{array}$$

27T. Tell time to the nearest half-hour.

28D. Add two-digit and three-digit numbers, no regrouping necessary.

29D. Add a one-digit number to a two-digit number with regrouping.

30C. Recognize that numerals such as 57 can be expressed as $40 + 17$.

30D. Add a one-digit number to a three-digit number with regrouping.

30Q. Recognize that there is only one line through two points.

31C.-(A) Use sentences like $27 = 20 +$
31C.-(B) 7 and $43 = 30 + 13$ to indicate regrouping or the use of different symbols for the same number.

31D. Add two-digit numbers with regrouping.

31E. Subtract a two-digit number from a two-digit number without regrouping.

31Q. Recognize that two lines can intersect at only one point.

The student should be able to:

32C.-(A) Demonstrate an understand-
32C.-(B) ing of grouping and re-grouping by completing sentences such as 458 = __ + 50 + 8 and 394 = 3 hundreds + __ tens + 4 ones.

32D. Add numbers with four or more digits, without regrouping.

32E. Subtract a number with one or more digits from a number with three or more digits, without regrouping.

32Q. Recognize that many lines may pass through a point.

33D. Add numbers containing three or more digits, with regrouping.

33E. Subtract a one-digit number from a two-digit number with regrouping.

34D. Estimate the sum of two numbers. For example, 287 + 520 is approximately 300 + 500 or 800.

34E. Subtract a two-digit number from a two-digit number, with regrouping.

35A. Identify the process of multiplication through experience with joining several equivalent sets of objects.

35D. Find solutions for sentences like $\square$ + 239 = 239 + $\square$ and 1984 + ($\square$ + 548) = (1984 + $\square$) + 548 to generalize the idea of the commutative and associative properties of addition.

35E. Subtract a number with one or more digits from a number with three or more digits, with regrouping.

35T. Make change for any purchase up to $1.00.

36C. Write many names for the same number, such as 7 + 5, 4 x 3, 10 + 2, and 12 for twelve.

36F. Use the multiplication facts through the product 18.

36W. Use the number line to illustrate multiplication problems.

37C. Write four-digit numerals in expanded notation; for example, 4567 = 4000 + 500 + 60 + 7.

37F. Recognize one's special role in multiplication in the set of whole numbers.

37M.-(A) Use sentences like 3 x 5 =
37M.-(B) __, __ x 7 = 14, and 4 x __ = 12 to represent physical, pictorial and verbal situations and find solutions for the sentences.

37V. Identify various instruments of measurement of time, temperature, weight, and length, such as clocks, thermometers, scales, rulers.

38L. Place the correct symbol ($<$, $>$, =) in the placeholder in sentences such as 3 x 5 $\square$ 7 + 8, 25 + 42 $\square$ 87 - 28, and 65 - 39 $\square$ 5 x 7.

38M. Find solutions for sentences like $\square$ x $\triangle$ = 36.

The student should be able to:

38V. Use non-standard units of linear measure and liquid measure, such as a pencil or book for length, and a paper cup for liquid measure.

39A. Demonstrate the concept of division by partitioning a set into several equivalent subsets; for example, separate a set of 12 objects into sets of 3 objects.

39F.-(A) Use the commutative and
39F.-(B) associative properties of multiplication in the set of whole numbers; for example, 4 x 3 = 3 x 4 and (4 x 5) x 2 = 4 x (5 x 2).

39H. Identify 2/3 and 3/4 of a whole.

39L. Make up story problems to fit sentences like 3 x 5 = □, □ x 7 = 14, and 4 x □ = 12.

39V. Make and use a ruler with divisions showing whole units.

40E. Recognize the inverse relation between addition sentences and two subtraction sentences, such as 725 + 342 = 1067 and 1067 - 725 = 342 and 1067 - 342 = 725.

40F. Recognize the distributive property of multiplication over addition in the set of whole numbers; for example, 3 x (2 + 4) = (3 x 2) + (3 x 4).

40V.-(A) Use standard units to the
40V.-(B) nearest whole unit for
40V.-(C) linear measure (inches and feet), for weight (pounds), and for liquid measure (pints and quarts).

41F. Recall the multiplication facts through 10 x 10.

41Q. Recognize common plane figures and their properties (triangles, circles, curves, rectangles, polygons).

42J. Discover number patterns from the addition and multiplication tables.

42T. Tell time to the nearest quarter hour.

43B. Determine the cardinality of a set to 10,000 through appropriate experiences.

43H. Show 2/4 = 1/2, etc., by the use of physical objects or pictures.

43J. Determine the factors of a counting number (a whole number other than zero) less than 100.

44C. Use roman numerals through XXV.

44F. Use the vertical algorithm to carry out multiplication with one factor less than 10 when regrouping may be involved.

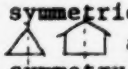
44J. Determine common factors of two counting numbers, each less than 100.

44N. Interpret simple ratio situations, such as 2 apples for 15¢, written $\frac{2 \text{ (apples)}}{15 \text{ (cents)}}$

44Q. Recognize rays and angles.

44R. Recognize that figures are similar if they have the same shape. For example, all squares are similar.

The student should be able to:

44S. Recognize symmetry with respect to a line by folding paper containing symmetrical figures such as  along their axes of symmetry.

45C.-(A) Write and read any numeral
45C.-(B) to 10,000.

45F. Multiply a one-digit number by 10, 100, 1000, . . ., .

45G. Demonstrate with sets of objects the relationship between such sentences as $4 \times 7 = 28$, $28 \div 4 = 7$ and $28 \div 7 = 4$.

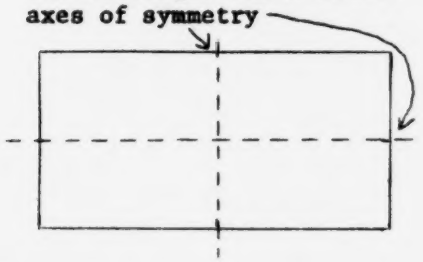
45H. Recognize greater than or less than for the fractions 1/4, 1/3, 1/2 with physical objects. (Note that fraction is being used here for rational number.)

45L. Use sentences like $9 \times 5 = 45$ and $45 \div 5 = 9$ to show division as the inverse of multiplication.

45M. Recognize that a sentence like $3 \times \square = 7$ has no whole number solution.

45N. Recognize the ratios such as $\frac{2 \text{ (pencils)}}{5 \text{ (cents)}}$ and $\frac{4 \text{ (pencils)}}{10 \text{ (cents)}}$ are equal ratios (represent the same ratio).

45S. Recognize that some figures have two or more axes of symmetry through paper folding. For example, two axes of symmetry are indicated for the rectangle shown below:



46C. Interpret place value to 10,000.

46F. Multiply a one-digit number by 10, 20, 30, . . . , 90.

46G. Recognize for each multiplication combination ($0 \times 0 = 0$ through $9 \times 9 = 81$) the corresponding division combinations, excluding division by zero. Examples: $7 \times 8 = 56$ has $56 \div 8 = 7$ and $56 \div 7 = 8$ as corresponding division combinations, but $0 \times 9 = 0$ has only one corresponding division combination, i.e. $0 \div 9 = 0$.

46K. Recognize that there is no largest whole number.

46L. Use parentheses to show order of operation; for example, $2 + (4 \times 3) = 14$ and $(2 + 4) \times 3 = 18$.

46N. Make up sets of equal ratios for given physical situations, such as {1/2, 2/4, 3/6, 4/8,. . .}

46V. Use common standard units such as inches, feet, yards, miles, in determining the measure of a distance.

47F. Multiply a two-digit number by 10, 100, 1000.

47G. Use repeated subtraction to do division computation.

47L. Use sentences like $36 \div 4 = \square$ and $\square \times 3 = 12$ to represent physical, pictorial and verbal situations.

The student should be able to:

47M. Find solutions for sentences like 36 ÷ 4 = ☐ and ☐ x 3 = 12.

47V.-(A) Use standard units of mea-
47V.-(B) sure, such as cups, gallons, ounces, in determining capacity and weight.

48F. Multiply a two-digit number by a one-digit number.

48G. Recognize the special role of 1 as a divisor.

48L. Make up story problems to fit sentences like 36 ÷ 4 = ☐ and ☐ x 3 = 12.

48W. Recognize that different (uniform) scales can be applied to the same line.

49F. Multiply a two-digit number by 10, 20, 30, . . . , 90.

49G. Divide a one-digit or a two-digit number by a one-digit number, with no remainders.

49L. Identify a mathematical sentence that represents a physical, pictorial and verbal situation involving more than one type of action.

49M. Find solutions for mathematical sentences involving more than one operation such as (2 x 5) + 4 = ☐ and (3 + 2) x ☐ = 10.

49W. Use the number line to illustrate division problems.

50F. Multiply a one-digit number by 100, 200, . . . , 900, 1000, . . . , 9000.

50G. Divide a three-digit number by a one-digit number, with no remainders.

50L. Make up story problems to fit a mathematical sentence involving more than one operation. For example, make up a problem situation to fit the sentence (3 + 4) x 2 = ☐ .

50V. Use standard units such as centimeters, meters, and kilometers in determining the measure of a distance.

50Z.-(A) Read and identify information
50Z.-(B) about a set of data presented in a table or a graph.

51F. Multiply a two-digit number by 100, 200, . . . , 900, 1000, . . . , 9000.

51N. Determine if two ratios are equal by using the property of proportions commonly called cross multiplication. For example, 3/4 = 9/12 because 3 x 12 = 4 x 9, whereas 6/7 ≠ 7/8 because 6 x 8 ≠ 7 x 7.

51V. Find the perimeter of a polygon.

52F. Multiply a number with three or more digits by 10, 100, 1000.

53D. Do column addition with several four-place or five-place addends with regrouping.

53F. Multiply a number with three or more digits by a one digit number.

The student should be able to:

53H. Construct sets of equal fractions through working with sets of objects. An example of such a set is $\{2/3, 4/6, 6/9, 8/12 \ldots\}$.

53W. Use the number line to represent positive rational numbers.

54F. Multiply a number with three or more digits by 10, 20, 30, . . . , 90.

54G. Divide a three-digit number by a one-digit number, with remainders.

54H. Write many names for the same rational number.

55C. Read and write any numeral to 10,000,000.

55F. Multiply a number with three or more digits by 100, 200, . . . , 900, 1000, . . . , 9000.

56C. Interpret place value to 10,000,000.

56F. Multiply a two-digit number by a two-digit number.

56G. Divide a number with two or more digits by 10, 20, 30, . . . , 90, with or without remainders.

56V. Estimate distances to the nearest unit.

57F. Multiply a number with three or more digits by a two-digit number.

57G. Divide a number with three or more digits by 100, 200, . . . , 900, 1000, . . . , 9000, with or without remainders.

57J. Identify prime numbers such as 2, 3, 5, 7, 11, 13, 17, . . .

57V. Recognize that all measurement involves approximation.

58F. Estimate the product of two numbers. For example, 21 x 88 is approximately 20 x 90.

58G. Estimate the quotient of two numbers. For example, 795 ÷ 23 is approximately 800 ÷ 20.

58J. Identify composite numbers.

59F. Find solutions for sentences like 723 x $\square$ = $\triangle$ x 723 to generalize the idea of commutative property of multiplication.

59G. Divide a number with two or more digits by a two-digit number, with no remainders.

60F. Find solutions for sentences like $\square$ x (176 x 19) = ($\triangle$ x 176) x 19 to generalize the associative property of multiplication.

60P. Identify faces, edges, vertices, and diagonals of common polyhedra.

60Q. Recognize isosceles and equilateral triangles and parallelograms.

60R. Recognize congruent segments as segments having the same length.

The student should be able to:

60V. Measure perimeters of triangles and quadrilaterals using English and metric units of measure.

61F. Find solutions for sentences like $3 \times 13 = (\square \times 10) + (\triangle \times 3)$, $\triangle \times \square = (5 \times 7) + (5 \times 8)$, and $\square \times 16 = (\triangle \times 10) + (\square \times 6)$ to generalize the distributive property of multiplication over addition.

61G. Divide a number with two or more digits by a two-digit number, with remainders.

61P. Identify the properties of various polyhedra by making appropriate paper models.

61R. Recognize congruent angles.

61V.-(A) Compare measures such as:
61V.-(B) 25 inches and 2 feet; 31
61V.-(C) ounces and 2 pounds; 75
61V.-(D) seconds and 1 minute; and 15 pints and 2 gallons.

62C. Use simple exponents such as $10^2 = 100$, $3^2 = 9$, and express 300 as 3×10^2.

62F. Multiply a number with three or more digits by a three-digit number.

62G. Divide a number with three or more digits by a three-digit number, with or without remainders.

62R. Recognize congruent plane figures as figures which have the same size and shape.

62V. Estimate perimeters of polygons, such as rectangles, triangles, and parallelograms.

63F. Use the abbreviated form of the multiplication algorithm with both factors having at least three digits.

63N. Use members of sets of equal ratio with the same first member or the same second member to compare different ratios. For example, to compare 5/9 and 3/4 show $20/36 < 27/36$, and to compare 5/11 and 3/7, show $15/33 > 15/35$.

63V. Compare perimeters of polygons, such as rectangles, triangles, and parallelograms.

64C. Determine greater than, less than, and betweenness for positive rational numbers.

64J. Find the prime factors of any number through 1000.

64N. Use cross products to compare different ratios such as $5/9 < 3/4$ because $5 \times 4 < 9 \times 3$. Likewise, $5/11 > 3/7$ because $5 \times 7 > 11 \times 3$.

65C. Use exponential notation in representing numbers; for example, $2345 = 2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 5$.

65G. Use the conventional division algorithm.

65N. Use the cross multiplication property as a first step in finding the solution of a proportion such as $2/5 = \square/9$.

66N. Find the missing whole number in two equal ratios like $2/3 = \square/9$ or $5/\square = 25/70$.

The student should be able to:

67H. Recognize that there is no smallest positive rational number.

67J. Determine the greatest common factor of two counting numbers.

67N. Use equal ratios to convert units of measure, such as (gallons) $\frac{1}{8} = \frac{3}{\square}$ (pints).

68C. Use bases other than 10 to demonstrate an understanding of base and place of a numeration system. (Computation should not be stressed.)

68J. Determine the least common multiple of two counting numbers.

68L. Write mathematical sentences using fractions to represent physical, pictorial and verbal situations.

69C. Find many ways to express a positive rational number: for example,

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \text{ and}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}.$$

69K. Use negative numbers to describe physical, pictorial and verbal situations.

69M. Use all of the previously introduced sentence forms with fractions.

69Q. Recognize a plane as a flat surface which contains lines and points.

69W. Use the number line to represent negative integers.

70D. Add simple rational numbers by the use of physical objects, diagrams, etc.

70E. Subtract simple rational numbers by the use of physical objects, diagrams, etc.

70F. Multiply simple rational numbers by the use of physical objects, diagrams, etc.

70P. Interpret space as the set of all points.

70Q. Interpret a circle as the set of all points in a plane that are at the same distance from a fixed point.

71Q. Recognize parallel lines as lines in a plane which do not intersect.

72D. Add rational numbers whose fractions have like denominators (not including conversion to mixed numerals). Examples: 2/7 + 3/7, 3/2 + 7/2.

72E. Subtract rational numbers whose fractions have like denominators.

72Q. Recognize the radius and diameter of a circle.

73C. Read and write simple decimals.

73D. Recognize that zero is the identity element for addition in the set of positive rational numbers as well as in the set of whole numbers.

The student should be able to:

73E. Recognize that subtraction is not always possible in the set of positive rational numbers.

73Q. Recognize perpendicular lines.

73R. Recognize that all congruent figures are similar, but not all similar figures are congruent.

74D. Add rational numbers whose fractions have unlike denominators.

74F. Multiply rational numbers expressed in fraction form.

74Q. Recognize acute, right, and obtuse angles.

75E. Subtract rational numbers whose fractions have unlike denominators.

75F. Recognize that 1/1 or 1 is an identity element for multiplication in the set of rational numbers.

75G. Express the quotient of positive integers as a mixed numeral; for example, $24 \div 5 = 4\ 4/5$.

75R. Recognize that triangles are congruent if corresponding sides are congruent and corresponding angles are congruent.

75X. Reproduce a line segment by using a compass and straightedge.

76G. Recognize that the operation of division is the inverse of multiplication in the set of positive rational numbers. For example, the sentences $3/4 \times 2/3 = 1/2$, $1/2 \div 3/4 = 2/3$ and $1/2 \div 2/3 = 3/4$ have this relationship.

76X. Interpret simple picture bar and line graph.

77D. Add rational numbers expressed in mixed numeral form. Example: $1\ 2/3 + 2\ 5/7$.

77F. Multiply rational numbers expressed in mixed numeral form.

77G. Recognize the multiplicative inverse (reciprocal) for every positive rational number except zero and use it in the division of rational numbers. For example, $1/2 \div 3/4 = 1/2 \times 4/3$.

77K. Recognize that the integers (positive and negative whole numbers and zero) are an extension of the whole numbers.

77V. Measure an angle by using a protractor.

77X. Construct simple picture, bar and line graphs.

78E. Subtract (without regrouping) rational numbers expressed in mixed numeral form.

78F. Use the commutative and associative properties of multiplication for positive rational numbers.

The student should be able to:

78G. Divide rational numbers expressed in fractional form.

78K. Find the additive inverse (opposite) for each integer by using the number line.

78V. Recognize that the sum of the measures of the angles of a triangle is 180°.

79E. Subtract (with regrouping) rational numbers expressed in mixed numeral form.

79G. Divide rational numbers expressed in mixed numeral form.

79K. Determine greater than, less than and betweenness for integers.

79R. Recognize the similarity of maps made with different scales.

79V. Recognize that a right angle has the measure 90°.

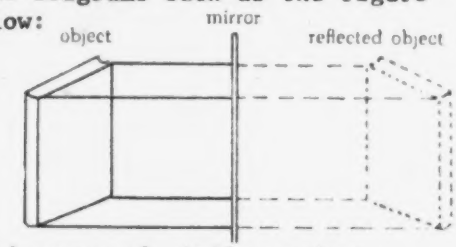
80C. Rename a decimal as a fraction.

80D.-(A) Use the associative and
80D.-(B) commutative properties for addition in the set of positive rational numbers.

80F. Use the distributive property of multiplication over addition with respect to positive rational numbers.

80G.-(A) Rename a fraction as a
80G.-(B) decimal, (terminating, repeating).

80S. Recognize the reflection of a plane figure in a mirror and draw diagrams such as the figure below:



80V. Find areas of simple regions informally. For example, a rectangular region with dimensions 2" by 3" can be covered by six one-inch squares (regions).

80W. Recognize that points in a plane (the first quadrant) can be represented by (ordered) pairs of numbers (coordinates).

81D.-(A) Use previously described
81D.-(B) sentence forms to generalize the commutative property of addition and the associative property of addition for positive rational numbers in fraction form.

81F. Use previously described sentence forms to generalize the distributive property of multiplication over addition for positive rational numbers in fractional form.

81N. Use equal ratios to convert fractions to decimals and conversely; for example, to write 3/5 as hundredths, solve for n in 3/5 = n/100; to write 44 hundredths as 25ths, solve for n in the sentence n/25 = 44/100.

81P. Demonstrate through paper folding an understanding of a line as an intersection of two planes.

The student should be able to:

81Q. Recognize the properties of isosceles triangles, equilateral triangles, and scalene triangles; such as, the longest side of a triangle is opposite the angle of greatest measure.

82D. Add rational numbers expressed in "terminating" decimal form.

82E. Subtract rational numbers expressed in terminating decimal form.

82L. Recognize a number sentence involving decimals that represents a given physical, pictorial, and verbal situation.

82N. Use the ideas of ratio and equal ratios with problems that include fractions as terms. For example, find the missing number in $\frac{2/3}{5} = \frac{\square}{20}$.

82P. Describe lines as intersections of planes.

82Q. Recognize that a plane is determined by three points not all on one line.

83F. Multiply rational numbers expressed in terminating decimal form.

83L. Write a mathematical sentence involving decimals that can be used to solve a given problem.

83N. Interpret percent as a ratio in which the second term is always 100.

83P.-(A) Make models of various prisms
83P.-(B) and find their surface areas.

84G. Divide rational numbers expressed in terminating form.

84N. Solve all three cases of percentage problems as problems in which you find the missing term of two equal ratios. For example, 20% of 30 and: $20/100 = \square/30$; 30 is what percent of 55 and: $\square/100 = 30/55$; 25 is 40% of what number and: $40/100 = 25/\square$.

84P. Recognize that a line (one-dimension) is a subset of a plane (two-dimensions) and that both are subsets of space (three-dimensions).

84W. Use ordered pairs of rational numbers to represent points in a plane.

85F. Use previously described sentence forms to generalize the commutative property of multiplication for rational numbers in any form.

85G. Divide rational numbers expressed in repeating decimal form.

85N. Solve ratio problems where some or all of the terms of the ratios are written as decimals.

85P. Recognize parallel planes.

86F. Use previously described sentence forms to generalize the associative property of multiplication for rational numbers in any form.

The student should be able to:

86M. Use all of the previously described sentence forms with decimal numerals.

86V. Estimate the area of an irregular plane region by use of a grid where an approximation to the area is the average of the inner and outer areas.

87C. Recognize that finding an integral power of a number involves repeated multiplication of the same number. For example, $(2/3)^3 = 2/3 \times 2/3 \times 2/3$.

87F. Use previously described sentence forms to generalize the distributive property of multiplication over addition for rational numbers in any form.

87N. Use proportions in problems about the lengths of sides of similar triangles.

87V. Recognize informal concepts of volume; for example, a box with dimensions 2" by 3" by 4" contains 24 one-inch cubes.

88D. Add integers.

88V. Find the volume of a rectangular prism.

88X. Bisect a line segment by using a compass and straightedge.

89C. Recognize that there is no smallest or largest rational number between two positive integers.

89D. Use the commutative property of addition for integers.

89X. Bisect an angle by using a compass and straightedge.

90C. Recognize that the rational number system is dense; that is, between each two different rational numbers, there is another rational number.

90X.-(A) Reconstruct a triangle and
90X.-(B) an angle by using a compass and a straightedge.

91A. Determine whether two sets are equivalent.

91C. Express large numbers by using scientific notation such as the distance from earth to the sun as 9.3×10^7 miles.

91V. Use formulas to compute the areas of rectangular, triangular and parallelogram regions.

91X. Construct a line perpendicular to a given line by using a compass and straightedge.

92A. Determine whether two sets are equal.

92P. Recognize common polyhedra, such as tetrahedron, a cube, a rectangular prism.

92V. Use formulas to compute the volume for common solids.

92X. Construct parallel lines by using a compass and straightedge.

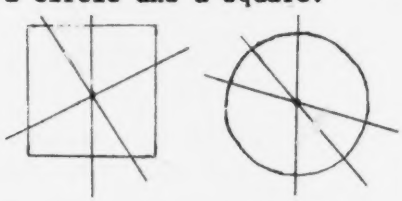
93A. Determine whether an element is a member of a given set.

93V. Use the metric system of measure for length and area.

The student should be able to:

94A. Given a universal set describe or list the members of a given set. For example, in the universal set of counting numbers the set of multiples of 3 is $\{3, 6, 9, 12 \ldots\}$.

94S. Recognize symmetry with respect to a point by folding paper along a line through the center of such geometric figures as a circle and a square.



94V. Recognize the relationship between the circumference and the diameter of a circle.

95A. Determine whether a set is a subset of a given set.

95V. Work with approximate numbers. For example, know that the area of a rectangle whose sides measure 6.5 and 3.6 inches to the nearest tenth of an inch has an area between 6.4 x 3.5 and 6.6 x 3.7 square inches.

96A. Recognize that the empty set is a subset of every set.

96V. Use the formula to compute the circumference of a circle.

97A. Identify the elements in the intersection of two sets.

97C. Given a set of numerals, classify them as representing whole numbers, and/or integers, and/or rational numbers. (cont.)

Examples: 8/4 represents a whole number and an integer and a rational number .333 . . . represents a rational number; -4 represents an integer and a rational number.

97D.-(A) Recognize and apply the
97D.-(B) addition properties of the
97D.-(C) system of positive ration-
97D.-(D) al numbers. (closure,
97D.-(E) identity, inverse, commutative, associative)

97F.-(A) Recognize and apply the
97F.-(B) multiplication properties
97F.-(C) of the system of positive
97F.-(D) rational numbers. (clo-
97F.-(E) sure, identity, inverse, commutative, associative)

97H. Interpret a fraction as a part of a whole, as expressing a ratio, as part of a group, or as an indicated division.

97J. Express a counting number in prime factor form. For example, $24 = 2 \times 2 \times 2 \times 3 = (2^3)(3)$.

97M. Establish the truth value of simple mathematical sentences. (Examples: $3 + 4 = 8$ is false; $35 \times 24 < 30 \times 20$ is false; $13 - 7 \neq 5$ is true.)

97N. Recall that a ratio can be used to compare quantities as well as to express a rate.

97Q. Describe, in terms of set, subset, union and/or intersection of sets of points, the following:

Half plane	Intersecting lines
Angle	Parallel lines
Triangle	Interior regions
Polygon	Etc.

97R. Recognize that triangles are congruent if corresponding sides are congruent and corresponding angles are congruent.

The student should be able to:

97S. Identify symmetry with respect to a line.

97V. Use various measuring instruments (such as rulers having English and metric scales, protracters, calipers) and record results to the nearer smallest unit of the scales. Example: Use rulers calibrated in tenths of an inch, in sixteenths of an inch, in centimeters and millimeters, etc.

97Y. Determine equality of two given ordered pairs of numbers. Example: $(2, 3) \neq (3, 2)$.

97Z. Organize and present data using a frequency table and graphs.

98A. Identify the elements in the union of two sets.

98C. Use expanded notation in representing whole numbers. (Examples: $314_{seven} = 3 \times 7^2 + 1 \times 7 + 4 \times 1$, and $314_{ten} = 3 \times 10^2 + 1 \times 10 + 4 \times 1$.)

98D. Recognize and apply the addition properties of the system of integers.

98F. Recognize and apply the distributive property in the system of positive rational numbers.

98H. Recognize that rational numbers can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

98J. Apply divisibility tests for 2, 3, 4, 5, 6, 8, and 9. Example: 135 is divisible by 3 because the sum of its digits is divisible by 3, by 5 because the last digit is 5, and by 9 because the sum of its digits is divisible by 9.

98N. Solve all types of percentage problems as problems in which they find the missing term in a proportion. (Including problems involving percents less than one and greater than one hundred.)

98Q.-(A) Classify angles (right, acute,
98Q.-(B) obtuse); recognize supplementary and complementary angles.

98R. Identify corresponding parts of congruent plane figures.

98V. Estimate linear measurements without using measuring instruments. Example: The length of a building, the width of a street, the height of a flagpole.

98W. Add and subtract integers by using the number line.

98X. Construct an equilateral triangle given one side using a compass and a straightedge.

98Y. List the members of a Cartesian product such as A X B where Set A has elements 1, 2 and 3, and Set B has elements 2, 4, -6, and 0, and graph A X B. (A X B is read "A cross B").

The student should be able to:

98Z. Read and interpret the information about a set of data presented in the from of a frequency table or a graph.

99A. Given a subset of a universal set identify the members of its complement.

99C. Write any rational number in decimal notation.

99H. Recognize that one and only one of the following statements is true when a and b are any two rational numbers: $a < b$, $a = b$, $a > b$.

99J. Using prime factorization find the least common multiple (LCM) and greatest common divisor (GCD) of two whole numbers and use them in computation with non-negative rational numbers.

99P. Classify prisms and pyramids according to their bases.

99Q. Classify sets of polygons (quadrilaterals, rectangles, squares, rhombuses, parallelograms, trapezoids, pentagons, hexagons, etc.).

99R. Recognize that corresponding parts of congruent plane figures are congruent, ($\cong$).

99V. Determine the sum of the measures of the angles of a quadrilateral.

99X. Construct the perpendicular bisector of a line segment using a compass and a straightedge.

99Y. Calculate the number of elements in a Cartesian product given the number of elements in each set.

99Z. Distinguish between certain and uncertain events.

100C. Use scientific notation to express numbers greater than ten.

100D.-(A) Add non-negative rational
100D.-(B) numbers in decimal form. (terminating, repeating)

100E.-(A) Subtract non-negative rational
100E.-(B) numbers in decimal form. (terminating, repeating)

100H. Determine, given two rational numbers, which is greater.

100M. Find the solution sets of simple open sentences using as a universe the whole numbers. (Example: $3N + 4 = 16$.)

100Q. Identify regular polygons.

100R. Recognize that circles are congruent if they have congruent radii.

100V. Determine the perimeters and areas of regular polygons.

100X. Make models of pyramids.

100Y. Identify relations such as parallelism, perpendicularity, equality, not-equal-to, congruence and similarity.

100Z. Recognize when the outcomes for a given experiment are equally likely. For example, when flipping a fair coin the outcomes "head" and "tail" are equally likely.

The student should be able to:

101C. Use positive integral exponents to express the power of a positive rational number.

101F. Multiply non-negative rational numbers in decimal form.

101G. Divide non-negative rational numbers in decimal form, recognizing that division by zero is undefined (impossible).

101H. Determine, given three rational numbers, which one is between the other two.

101R. Identify corresponding parts of similar plane figures.

101V. Use the formula to compute the area of a circle.

101X. Construct a rhombus and a square given one side using a compass and a straightedge.

101Y. Identify relations such as is-a-factor-of, less than, greater than, is-a-subset-of, is-an-element-of, is-the-square-of, etc., and formulate a set of ordered pairs of elements from each. Example: Using the relation is-a-factor-of in the following set of numbers, $\{1, 2, 3, 6\}$ formulate the set of ordered pairs identifying the relation. Answer: $\{(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)\}$.

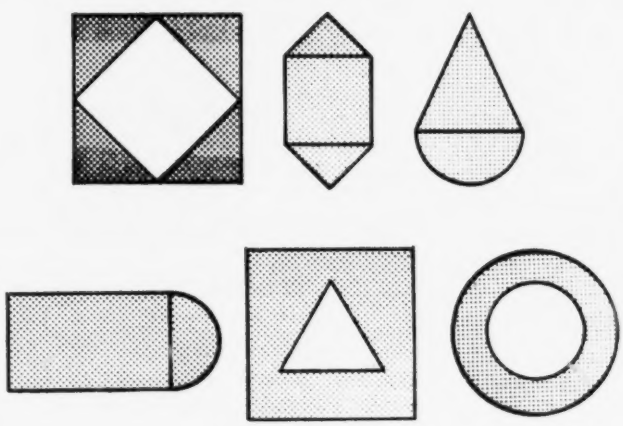
101Z. List and/or count all the possible outcomes for an experiment for which the simple events are single elements. For example, spinning a spinner, tossing a die, drawing a lot, flipping a coin.

102G. Compute positive integral powers of non-negative rational numbers. (Examples: $3^4 = ?$; $1/2^3 = ?$).

102H. Compute the mean (arithmetic average) of a set of non-negative rational numbers.

102R. Recognize that corresponding angles of similar plane figures are congruent.

102V. Determine the areas of figures such as those shown below:



102W. Graph, using the number line, solution sets for equalities and inequalities where the replacement set is the set of integers, such as $X > -2$; $X + 3 < 12$; $X > -3$ and $X < 5$; $X > 5$ and $X < 4$; $X > -2$ or $X < 3$; $X < 5$ or $X < 3$; $X > 4$ or $X < -2$.

102Z. Assign probabilities to the outcomes of an experiment for which the simple events are equally likely. For example, the probability of tossing a "2" with one toss of a die is 1 chance out of 6 or 1/6.

The student should be able to:

103H. Given any two rational numbers, find a rational number that is between them.

103R. Recognize that radii, diameters, corresponding altitudes and corresponding diagonals of congruent plane figures are congruent.

103V. Determine the surface area and volume of prisms and pyramids.

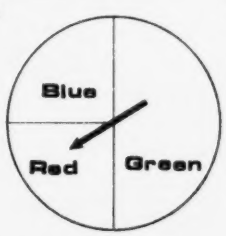
103W. Use ordered pairs of rational numbers as coordinates of points in a plane. Example: (2,3), (-5,7), (-2/3, -3/8).

103Y. Graph relations (including functions in the planes) W X W (whole numbers), I X I (integers), and Q X Q (rationals).

103Z. Recognize that the probability of an event is a number p such that $0 \leq p \leq 1$, and recognize that the probability of a certain event is 1 and the probability of an impossible event is 0.

104V. Determine greatest possible error for any calibration of a given measuring instrument.

104Z. Assign probabilities to the outcomes of experiments for which the simple events are not equally likely. For example, the probability of the arrow stopping on red on the spinner at the right is 1 chance out of 4 or 1/4.



105V. Recognize the relationship between precision and greatest possible error.

105Z. Recognize that the probability of an event does not guarantee how often the event will occur. For example, the probability of obtaining a "2" when a fair die is tossed is 1/6; however, this does not mean that the "2" will occur once out of every six tosses.

106A. Determine the number of subsets in a set containing N elements. For example, a set containing 5 elements has 32 or 2^5 subsets.

106C. Show that a rational number can be expressed as a repeating decimal, and that a repeating decimal can be expressed in fractional form.

106M. Define equivalent open sentences as sentences having the same solution set.

106P. Identify spheres, circular cylinders, and circular cones.

106Q. Describe, in terms of set, subset, union and/or intersection of sets of points, the following:

Skew lines	Central angle
Tangent, secant	Arc
	Diagonal

106R. Recognize that two triangles are congruent if two sides and the included angle of one are congruent to the corresponding parts of the second (SAS).

106S. Identify symmetry with respect to a plane.

The student should be able to:

106V. Determine the relative error given a measuring instrument and an object to measure.

106X. Given three sides, or two sides and the included angle, or two angles and the included side, construct the triangle.

106Z. Estimate the probability of an outcome of an experiment by empirical methods. For example, estimate the probability of a bottle cap landing with the cork side up when it is flipped (the student can flip a bottle cap 100 or more times, keeping a record of how the cap lands, and estimate the probability from his data).

107C. Identify non-repeating decimals representing irrational numbers. For example, .232332333233332 . . . .

107E. Define subtraction in terms of addition in the systems of integers and rational numbers. (Example: a - b means a + (-b).)

107M. Find solution sets of open sentences by finding equivalent open sentences using the properties of equality.

107P. Identify regular solids.

107Q. Recognize the various plane figures formed when a plane intersects a given figure such as a square pyramid or a cube.

107R. Recognize that two triangles are congruent if two angles and the included side of one are congruent to the corresponding parts of the second (ASA).

107V. Recall the relationships between units of the metric system.

107X. Determine whether you can construct none, one, two, or many triangles given three angles or given two sides and an angle not included between them.

107Z. List and/or count all the possible outcomes for an experiment for which the simple events are ordered pairs, simple combinations, or simple permutations. For example, flipping two coins, tossing a pair of dice, rearranging the order of three letters of the alphabet, spinning a spinner and simultaneously tossing a coin, etc.

108C. Use positive, zero, and negative exponents correctly.

108D. Add integers and rational numbers.

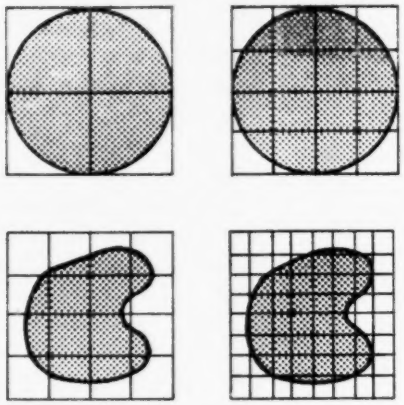
108E. Subtract integers and rational numbers.

108Q. Recognize convex and nonconvex polygons.

108R. Recognize that two triangles are congruent if the three sides of one are congruent to the corresponding three sides of the second (SSS).

The student should be able to:

108V. Improve results obtained by estimating the area of plane figures through the use of grids by using grids containing smaller units. Examples:



108Z. Assign probabilities to the outcomes of an experiment for which the simple events are ordered pairs, combinations or permutations. For example, the probability of getting 2 heads when 2 coins are flipped simultaneously is 1 out of 4 or 1/4.

109C. Use scientific notation to express positive rational numbers; for example, the length of a microwave is 2.2×10^{-2} meters.

109F.-(A) Multiply integers and rational
109F.-(B) numbers. (positive and both positive and negative)

109G. Divide integers and rational numbers.

109R. Recall that the measures of radii, diameters, corresponding altitudes, and corresponding diagonals of similar plane figures are proportional.

109V. Determine the sum of the measures of the angles of regular polygons.

109W. Use ordered pairs of real numbers as coordinates of points in a plane. Example: $(2,3)$, $(-5,7)$, $(-1/2,-3)$, $(\sqrt{2},-8)$.

109Y. Graph relations (including functions) in the planes I X I (integers), Q X Q (rationals), and R X R (reals).

109Z. Recognize how the probability of a specific outcome will change from trial to trial when sampling is done without replacement. For example, from an urn containing 3 red marbles and 2 white marbles the probability of drawing a red marble is 3/5, but the probability of drawing a red marble if the first is not replaced then is either 2/4 or 1/4.

110C. Use the symbol $\sqrt{a}$ to indicate one of two equal factors whose product is a.

110H. Compute the mean of a set of rational numbers.

110M. Find solution sets of simple open sentences using as a universe the integers. (Examples: $5N + 4 = -16$.)

110N. Use proportions to solve problems involving similar triangles.

110R. Recognize that triangles are similar if corresponding angles are congruent.

The student should be able to:

110V. Recall the relationship between degrees, minutes and seconds as units of angular measurement.

110Z. Estimate the probability of an event from previous data. For example, a basketball player made 50 free throws out of 75 tries; what is the probability that he will make his next free throw attempt?

111C. Use expanded notation to express positive rational numbers. (Example: $21.37 = 2 \times 10^1 + 1 \times 10^0 + 3 \times 10^{-1} + 7 \times 10^{-2}$)

111R. Recognize that triangles are similar if the measures of corresponding sides are proportional.

111V. Do computation with measures (approximate numbers) and determine the greatest possible error and the relative error of each result. (Addition, subtraction, multiplication, division)

111W. Graph using the Cartesian plane I X I (integers) solution sets of inequalities and equalities such as $x + y \neq 19$; $x + y > 19$; $x + y < 19$; $x + y = 19$.

111Z. Estimate information about a population by random sampling. For example, out of 100 flash bulbs selected at random 5 were defective. How many out of 3000 are likely to be defective?

112C. Compute products and quotients of numbers expressed in exponential notation (including scientific notation).
Examples:
$3^7 \cdot 3^5 = 3^{12}$; $a^5 \div a^2 = a^3$;
$(3 \times 10^2)(2.3 \times 10^3) = 6.9 \times 10^5$.

112D. Recognize and apply the addition properties of the rational number system.

112F. Recognize and apply the multiplication properties and the distributive property of the rational number system.

112K. Approximate square roots of positive integers.

112M. Perform a series of operations in proper order when grouping symbols are omitted, i.e., multiplications and divisions are performed first, in left-to-right order, then additions and subtractions are performed in left-to-right order.

112R. Recall that the ratio of the measures of two sides of a triangle is the same as the ratio of the measures of the corresponding two sides of a similar triangle.

112V. Use the Pythagorean relationship to determine the length of any side of a right triangle given the lengths of the other two sides.

113C. Determine, given a rational number and an irrational number, which is greater.
For example: which is greater, 2 or $\sqrt{3}$?

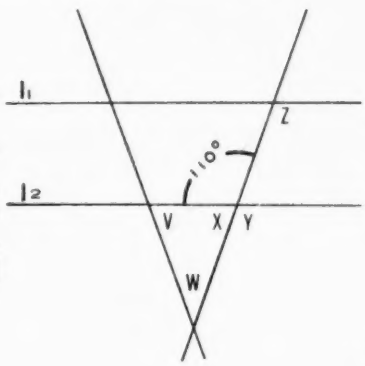
The student should be able to:

113F. Recognize and apply the multiplication properties of the system of integers.

113Q. Use relationships such as those involved in finding the solution of the following:

l_1 is parallel to l_2

Find the number of degrees in $<x$, $<y$, $<z$ and the sum of the measures of $<v$ and $<w$.



(Relationships involved: vertical angles, supplementary angles, alternate interior angles, exterior angle to remote interior angles of a triangle)

113R. Recall that sine, cosine and tangent ratios are independent of the measure of the sides of the triangles involved.

113X. Construct representations of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, etc.

113Y. Given a domain (such as a subset of the integers) and given a description of a relation, determine the set of ordered pairs and graph the relation. Example: $\{-3,-2,-1,0,1,2,3\}$ and the description of the relation is "double the number and add 3." The determined set of ordered pairs should be $\{(-3,-3), (-2,-1), (-1,1), (0,3), (1,5), (2,7), (3,9)\}$.

114V. Solve problems involving indirect measurement using the sine, cosine, and tangent ratios. (Including use of tables)

114Y. Determine whether a given relation is or is not a function. Examples:
$\{(a,b) \mid b=a^2, a,b \in R\}$ is a function, but
$\{(a,b) \mid a=b^2, a,b \in R\}$ is not a function.

115V. Use formulas to calculate volumes and surface areas of spheres, cylinders and cones.

115Y. Given the graph of a linear function, determine a set of ordered pairs which are members of the function and describe the function.

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